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Gampaha Education Zone

දෙවන වාර ඇගයීම - 2025
Second Term Evaluation - 2025
இரண்டாம் தவணைப் பரீட்சை - 2025

ශ්‍රේණිය Grade **12**	විෂයය Subject **Combined Mathematics I**	කාලය Time **2h 40 min**
නම பெயர் **Name**		

Instructions:

- This paper consists of two parts.
  Part A (Questions 1-8) & **Part B (Questions 9-12)**
- **Part A**
  Answer **all** the questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
- **Part B**
  Answer **all** the questions. Write your answers on the sheets provided.

- At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.
  You are permitted to remove **only Part B** of the question paper from the examination hall.

For Examiners' Use Only.

(10) Combined Mathematics I		
Part	**Question No.**	**Marks**
A	**1**	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
B	**9**	
	10	
	11	
	12	
	Total	

Total

In words	
In numbers	

Code Numbers

Marking Examiner	
Checked by	
Superviced by	

Part A

1. Let $f(x) = \frac{2x}{3x-1}$; $x \neq \frac{1}{3}$ is a one-to-one function. Find the $f^{-1}(x)$. Write the range of $f(x)$.
   Show that $f^{-1}\,[2f(1)] = \frac{1}{2}$

2. Find all real values of x satisfy the inequality $1 \leq \frac{3x}{x^2-4}$

3. Draw the graph of $y = |x-2|$ and $y = k - 2|x+1|$ in a same diagram, Such that the set of solutions satisfied by $|x-2| \leq k - 2|x+1|$ is $x \in \left[\frac{-4}{3}, 0\right]$, where $k \in Z^{+}$. Find the value of k.

4. Let $p(x)$ be a polynomial of x. It is given that when $p(x)$ is divided by $(x-1)$ and $(x-2)$, the remainders are 2 and 1 respectively. Find the remainder when $p(x)$ is divided by the polynomial $x^2 - 3x + 2$.

5. Let $f(x) = ax^2 + bx + c,\ a, b, c \in \mathbb{R}, a \neq 0$. The graph of $y = f(x)$ touches the x −axis at $x = \frac{1}{2}$ and cuts the y −axis at $y = 1$. Find the values of a, b and c.

6. Evaluate. $\lim_{x \to 0} \frac{\cos 3x - \cos 5x}{\sqrt{1+x^2} - \sqrt{1-x^2}}$.

7. Let $A(-1, 3)$ and $B(5, k)$ of which $AB = 10\ units$. $P(2, h)$ is a point on the line segment AB, according to a certain ratio. Find the positive real value of k. Also find the ratio $AP : PB$ and the value of h.

8. Solve the following equation.

$$\sin x + \cos x = sin\frac{5\pi}{12} + cos\frac{5\pi}{12}$$

Part *B*

9. a) Let $f(x) \equiv -2x^2 + \lambda x + \mu$
Write the function in the form of $f(x) \equiv -2(x+h)^2 + k$, where λ, μ, h and k are real constants.
If the coordinates of the turning point of $y = f(x)$ is (3,2) find the value of λ and μ.
For this values of λ and μ, sketch the graph of $y = f(x)$ by indicating the axis of symmetry , turning point and the y axis intercept.
Hence discuss the nature of the roots of the equation $2x^2 - \lambda x - \mu = 0$

Let $g(x) \equiv 4 - f(x+1)$.
Determine the equation of the axis of symmetry and the minimum value of $y = g(x)$.
Sketch the graph of $y = g(x)$
Hence discuss the nature of the roots of the equation $f(x+1) = 4$

b) Find the minimum and maximum value of m where $m \in \mathbb{R}$, such that the quadratic equation $(x+m+2)^2 + (x-m)^2 = 2$ has real roots.
Let α and β are distinct real roots of the above equation.
Write α + β and α β in terms of the coefficients of the equation.
Deduce that the sign of these two roots cannot be positive, for all values of m.
Also find the value of m such that $|\alpha - \beta| = 2$
Show that the quadratic equation whose roots are $3\alpha + 2$ and $3\beta + 2$ is $x^2 + 2x + 9m^2 + 18m + 1 = 0$

10. a) Let $f(x) \equiv 2x^3 - 7x^2 + 8x - k$, where $k \in \mathbb{R}^+$

It is given that $(2x - k)$ is a factor of $f(x)$. Find the values of k.

For $k = 3$, write $f(x)$ as a product of linear factors.
Deduce the roots of the equation $2x^3 - 7x^2 + 7x - 2 = 0$

b) Find the constant A, B and C such that $3x + 2 \equiv A(x-1)^2 + B(x^2 - x) + Cx$
Hence separate $\frac{3x+2}{x(x-1)^2}$ into partial fractions completely.
Deduce the partial fractions of $\frac{3x+5}{x^2(x+1)}$

c) Sketch the graphs of $y = 3|x-1|$ and $y = |x| + |x-1|$ in the same diagram.
Hence find the area of the region, which satisfy the inequality, $3|x-1| \le y \le |x| + |x-1|$

11. a) (i) Show that $\lim_{x \to a} \frac{sin^2 x - sin^2 a}{x - a} = \sin 2a$, where $a \in \mathbb{R}$

(ii). It is given that $x^2 - y + 7x - 6 = 0$

Evaluate $\lim_{x \to 1} \frac{\sqrt{x-1} - \sqrt{y-2}}{\sqrt{x^2 - 1}}$.

b) Find the greatest and the least value of the expression $\frac{x^2 - x + 1}{x^2 + x + 1}$, for all real values of x.

c) Solve the following simultaneous equations

$2^{x+3} - 8^{y+1} = 0$ and

$2\, log_9 (x) + log_3 (y) = 3.$

d) Let $A \equiv (3, 0)$ and $B \equiv (3, -2)$ and C is a point on extended BA line of which $BC{:}\,CA = 3{:}\,2$. Find the coordinates of the point C. Given that D is a point with coordinates $(6, 4)$.
Find AC, AD and CD lengths. Deduce what type of triangle is ADC.
Find the coordinates of the center of a circle, such that the point C, lies on the circumference of the circle. Further E is another point on the circumference, such that ACDE is a rectangle.
Find the coordinates of the point E.

12. a) Write an expression for $cos(A + B)$ in terms of $sinA, cosA, sinB$ and $cosB$.
Build up an expression for $\cos 5\theta$ in terms of $\cos\theta$.
Show that

$$\frac{cos5\theta}{cos\theta} = 16cos^4\theta - 20cos^2\theta + 5 , \qquad for\ cos\theta \neq 0$$

Hence show that the roots of the quadratic equation $16x^2 - 20x + 5 = 0$ are $cos^2\frac{\pi}{10}$ and $cos^2\frac{3\pi}{10}$.
Deduce that $sec^2\frac{\pi}{10} + sec^2\frac{3\pi}{10} = 4$

b) Let $f(x) \equiv \frac{1}{2}\left[\sin\left(x - \frac{\pi}{6}\right) + \sqrt{3}\right] + cos\, x$
Express $f(x)$ in the form of $f(x) \equiv K\,[sin(x + \alpha) + 1]$,
where $K > 0$ and α $[0 < \alpha < \frac{\pi}{2}]$ are constants to be determined.
Hence,

i) Show that , $f(x)$ is a non negative function.

ii) Solve the equation, $\frac{1}{2}\sin\left(x - \frac{\pi}{6}\right) + cos\, x = \frac{\sqrt{3}}{4}$

c) From the usual notation of a triangle, state the Sine rule and the Cosine rule for a triangle ABC.

If $cos\frac{A}{2} = \sqrt{\frac{\sin B + \sin C}{2 \sin C}}$, show that ABC is a right angled triangle.

Also show that, $a - b = \sqrt{2}\, c\, sin\left(\frac{A-B}{2}\right)$

d) Solve,

$2\, cos^{-1}x + cos^{-1} 2x = \pi$